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ELASTIC-PLASTIC ANALYSIS OF TUBE EXPANSION IN TUBESHEETS

B. Kasraie
J. S. Porowski
W. J. O'Donnell
A. Selz

O'DONNELL & ASSOCIATES, INC.
Pittsburgh, Pennsylvania

ABSTRACT

Conditions for expansion of tubes in tubesheets are often determined by the test. The tightness of the joint and pull out force are used as criteria for evaluation of the results. For closely spaced tubes, it is also necessary to control development of the plastic regions in the ligaments surrounding the tube being expanded. High local strains may occur and excessive distortion may result if the expansion of the tube is continued beyond the admissible limits.

Elastic-plastic finite element analyses are performed herein in order to establish conditions for rolling of the tubes in tubesheets of low ligament efficiency. Such penetration patterns are often required in the design of tubular reactors for catalytic processes. The model considered includes individual tube expansion in tubesheets with triangular penetration patterns. The effect of prior expansion of the neighboring tubes is also evaluated. Gap elements are used to model the initial clearance of the tube in the hole.

Development of the plastic zones and distortion of the ligaments is monitored during radial expansion of the tube diameter. The residual stresses between the tube and the hole surface and the history of gap closing after removal of the expansion tool are determined.

The effect of axial extension of the tube on the tube thinning is determined. Tube thinning is often used as a measure of tube expansion in manufacturing processes. For the analyzed ligament efficiency, reliable joints are obtained for a thinning range within 2% to 3%.

NOMENCLATURE

a radius of the tubesheet hole
d tube outer diameter
E modulus of elasticity
h minimum ligament width
H expansion length
P perforation pitch
P_A contact pressure between the tube and the tubesheet during plastic expansion
P_B elastically calculated contact pressure between the tube and the tubesheet
P_C inner pressure in the expanded tube
P_R residual contact pressure between the tube and the tubesheet
r radial coordinate in Figure 2
$S_y = \sigma_0$ yield strength
t thickness of the tube
ν Poisson's Ratio
σ stress
$\mu =$ ligament efficiency shown in Figure 4

INTRODUCTION

In large diameter reactors, heat exhangers, condensers or steam generators with fixed tubesheets, the tubes carry the axial pressure loads providing the support needed to reduce the thickness of the tubeshee Moreover, incompatible axial displacements of the shel and the tube bundle often cause significant additional axial forces in the tubes at the periphery of the perforated area of the tubesheet.

If no tube-to-tubesheet welds are provided, the rolling process must seal the gaps between the insertec tubes and the hole surfaces, as well as provide residual contact reactions which produce sufficient friction to support axial loads in tubes. When tube-to-tubesheet welds are provided, rolling is used to close gaps between the tubes and the penetrations to a degree such that the tube is well guided in the opening. Such a guidance eliminates the effect of tube bending on the welds, and the resulting friction reduces or eliminates axial loads imposed by the tubes on the welds.

For tubesheets with widely spaced holes, it is not too difficult to produce tight, strong joints. The joints can be sealed and sufficient friction is developed even for thin tubesheets where, for example, the tubesheet thickness remains within a range of the tube diameter. The process may be carried out as well by mechanical rolling or by hydrostatic expansion. For tube spacing where the ligament efficiency (see Figure 4) $\mu = h/p > 0.25$ the rolling process generally provides good results.

When the tubes are more closely spaced in order to reduce the size or increase the capacity of the unit, rolling is far less reliable. The thin ligaments may be easily overstretched during rolling. Expansion of the tubes should be limited in order to prevent plastic damage to the ligaments. The distribution of radial pressure along the hole boundary during expansion is nonuniform. The minimum ligament sections can bend toward the neighboring holes and thereby yield locally initiating cracks if the tube expansion is too large. Inadequate expansion results in joints which are not tight, and may not provide adequate pull-out strength. In order to determine the correct range of tube expansion, the elastic springback of the tube must be considered. When the yield strength of the tube material is significantly higher than that of the tubesheet material, the tube springback more than the penetration, causing the tubes to loosen. This makes it more difficult to fabricate a high integrity tube-to-tubesheet joint.

A brief discussion of the closed form analytical solution for a tube expanded in an infinite plate is given. This solution is used to explain how the residual contact pressure is raised by plastic deformation involved in the rolling process. Inelastic finite element solutions are used herein to analyze the expansion process. The analysis is performed for ligament efficiency $\mu \cong 13.2\%$, which allows for a tight packing of tubes desired in tubular catalytic reactors. Relatively thin wall tubes are considered since their use also allows better utilization of the expensive high pressure volume of the reactor.

The analysis is first performed for the expansion of a single tube in a perforated tubesheet with neighboring empty holes. This is the case where a good joint is most difficult to achieve. The strengthening effect of the tubes rolled into the neighboring holes prior to the tube being expanded is analyzed next. Finally, axial flow of the tube during expansion is investigated. The analyses include the effect of higher yield strength of the tube in comparison to the yield strength of the tubesheet. Practical recommendations concerning tube expansion are given based on the analytical results.

ANALYZED TUBE-TO-TUBESHEET JOINT

The geometry of the joint and the penetration pattern are shown in Figure 1. The dimensions and effective elastic constants of the pattern are as follows:

Tubesheet yield strength	50,000 psi
Tube yield strength	70,000 psi
Expansion length	$H = 50$ mm
Tube hole radius	$a = 19.3$ mm
Outer tube diameter	$d = 38.1$
Tube thickness	$t = 2.1$ mm
Penetration Pitch	$P = 44.5$ mm
Ligament efficiency	$\mu \cong h/p = 0.1325$
Effective Elastic Modulus	$E^*/E = 0.0856$
Effective Poisson's Ratio	$\nu^* = 0.5945$

BASIC SOLUTION FOR TUBE EXPANSION IN INFINITE PLATE

In order to generate residual contact pressure to hold the tube, the tubesheet material must be deformed plastically during the expansion process. A schemat of the solution for residual stresses in the tube-to tubesheet joint is shown in Figure 2. For simplicit the axisymmetric, plane problem of the tube expanded in the infinite plate is considered. As shown in th figure, the residual radial stress at each radial location is determined by the difference between the elastic and elastic-plastic solutions.

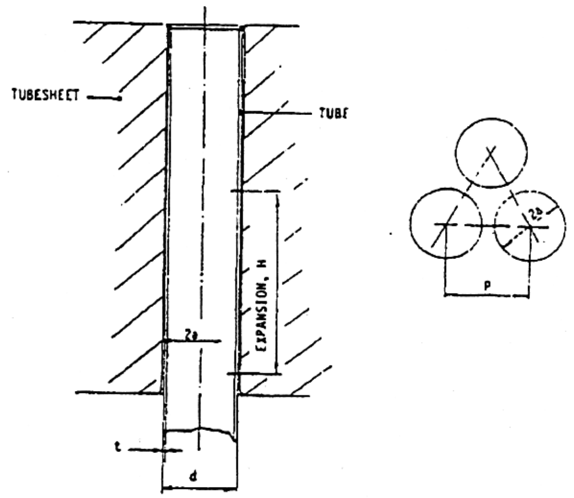

Figure 1 Geometry of the Joint and Penetration Patter

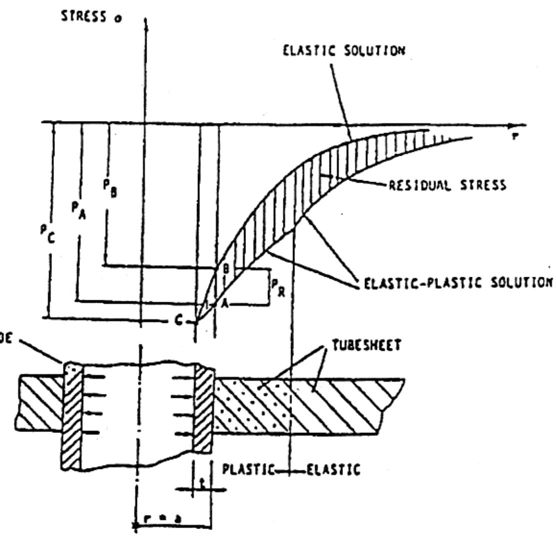

Figure 2 Use of Elastic and Elastic-Plastic Solutions for Evaluation of Residual Stresses

The residual contact pressure at the interface between the tube and the tubesheet is defined by the sector AB.

The elastic-plastic response of the plate with th radially expanded hole is given in Figure 3. The grap illustrates the gradual redistribution of radial stres with expansion of the plastic zone. As illustrated in Figure 2, the slope of the elastic curve for radial stresses through the tube wall increases faster with growing expansion pressure than the corresponding slope from the elastic-plastic solution. Thus the residual contact pressure determined by the difference between

This part of the analysis parallels the earlier Goodier & Schoessow work.
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elastic and plastic solutions also increases with progressing expansion of plastic zone.

Enlarging the hole up to the maximum value of the pressure $P_A = 1.155\ S_y$, where S_y is the yield stress of the material, improves the resulting residual contact pressure. Continuation of the hole enlargement beyond this limit causes only additional strains due to plastic flow of the material squeezed out in the axial direction.

The elastic solution for a pressurized hole in an infinite plate needed to obtain a springback response of the joint is simply:

$$\sigma_r = -\sigma_t = -P_C \frac{(a-t)^2}{r^2} \qquad (1)$$

Where σ_r and σ_t are radial and circumferential stresses respectively. Note that the equation is given for pressure P_C, and the inner radius $(a-t)$ is a numerator in the fraction.

The pressure drop through-the-wall for a perfectly plastic material is:

$$P_C - P_A = \frac{2S_y}{\sqrt{3}} \ln \frac{a}{a-t} \qquad (2)$$

is question - Note that the tube becomes fully plastic before closing the clearance in the hole.

For various pressures at the hole surface, the response of the plate is shown in the diagram of Figure 3. Equation (1) can be used to obtain the ratio of the internal to external pressures in the elastic solution for the tube. Using again the notation in Figure 2:

$$P_B/P_C = \left(\frac{a-t}{a}\right)^2 \qquad (3)$$

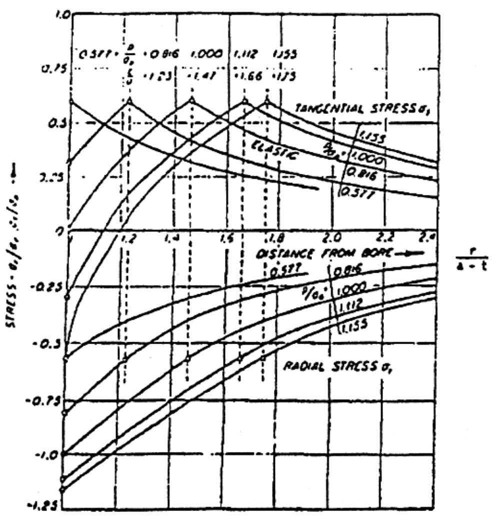


Figure 3 Stress Distribution in Steel Plate Having a Hole After Partial Yielding Through Radial Expansion [1]

Thus the residual stress at the interface expressed as a function of the pressure imposed by the expander on the inner surface of the tube is:

$$* \quad P_R = P_A - P_B = P_C\left[1-\left(\frac{a-t}{a}\right)^2\right] - \frac{2S_y}{\sqrt{3}}\left(\ln\frac{a}{a-t}\right) \qquad (4)$$

For the joint considered $a = 19.3$ and $t = 2.1$, thus:

$$** \quad P_C\left[1-\left(\frac{b}{a}\right)^2\right] - P_{exp}\, l_n\left(\frac{a}{b}\right)$$

$$P_R = P_C\left[1-\left(\frac{17.2}{19.3}\right)^2\right] - \frac{2S_y}{\sqrt{3}} \ln\frac{19.3}{17.2} = 0.206\ P_C - 0.133\ S_y$$

If, for example, expansion is continued until the pressure in the tube P_C reaches the yield strength of the material S_y, the residual stress is $0.073\ S_y$. This residual stress produces hoop compressive stress in the tube equal to

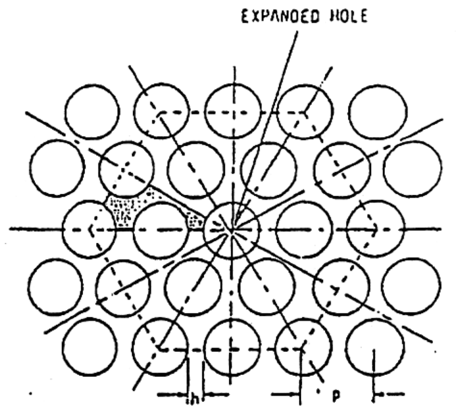
$$t = \frac{P_t a}{t} = 0.073\ S_y \cdot \frac{19.3}{2.1} = 0.671\ S_y \qquad (5)$$

As can be seen from Figure 3, the plastic regions in the expanded plate will reach to 1.47 of the inner radius of the tube $(a-t)$. The example indicates that large residual stresses can be generated in the tube expanded into a solid plate. The forthcoming analysis will indicate that much lower residual stresses result when the tube is expanded in the hole surrounded by the material weakened by neighboring penetrations.

FINITE ELEMENT ANALYSIS OF TUBE EXPANDED IN PERFORATED TUBESHEET

During expansion of the tube in the perforated plate, the radial enlargement of the tube is essentially independent of the orientation with respect to the penetration pattern. The stress distribution, however, is not axisymmetric as in the simple case considered previously. After expansion, the diameter of the expander is gradually reduced and the tube adjusts radially to maintain equilibrium. Its cross-section warps slightly and the resulting deformation is not axisymmetric.

The analyzed portion of the tubesheet is shown in Figure 4. Due to the symmetry of the triangular penetration pattern, only the solution for a 30° wedge of the pattern must be considered. The finite element model is restricted to the shadowed area. The reactions imposed by the remaining part of the sheet are simulated by an effective elastic material supporting the outer edge of the analyzed cell.



Ligament efficiency $\mu = h/P$

Figure 4 Analyzed Portion of the Perforated Sheet with Triangular Penetration Pattern

The finite element mesh is shown in Figure 5. A finer mesh is used in the vicinity of the expanded hole. The ANSYS Program was used to obtain elastic-plastic numerical solutions. The Model consists of 271 four-noded plane strain elements, 3 two-noded spring damper elements, and 32 two-dimensional interface (gap)

$* \frac{2}{\sqrt{3}} S_y = 1.155\ S_y$ - See ellipse in Yokell paper

** THIS PUTS THE EQUATION INTO TERMINOLOGY USED BY GOODIER & SCHOESSOW elements. Gap elements are used between the expander and the tube and also between the tube and the tube-sheet. These gap elements, when closed, are capable of supporting only compression in the direction normal to the surfaces. All gaps were initially open.

The boundary conditions used in the analysis are also shown in Figure 5. The inner gap elements were simultaneously closed and an axisymmetric radial displacement was imposed on the inner surface of the tube in small increments. After each load step, the solution was converged to satisfy the 1% convergence criteria recommended in ANSYS Program. The radial displacement was increased until cross plasticity of the ligament occurred. The joint was then elastically unloaded until all the gaps at the inner surface of the hole opened. Due to the presence of the gaps, the analysis continued to be nonlinear even though the tube and the tubesheet remained elastic.

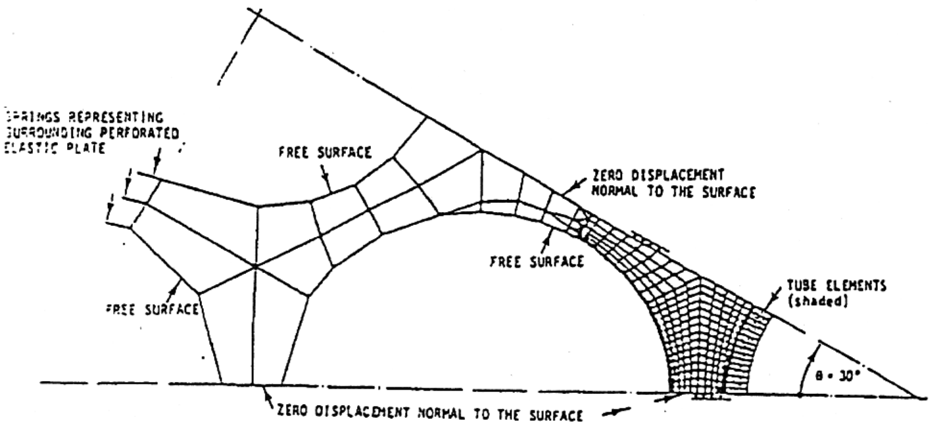


Figure 5 Finite Element Model Used To Analyze Expansion of a Single Tube

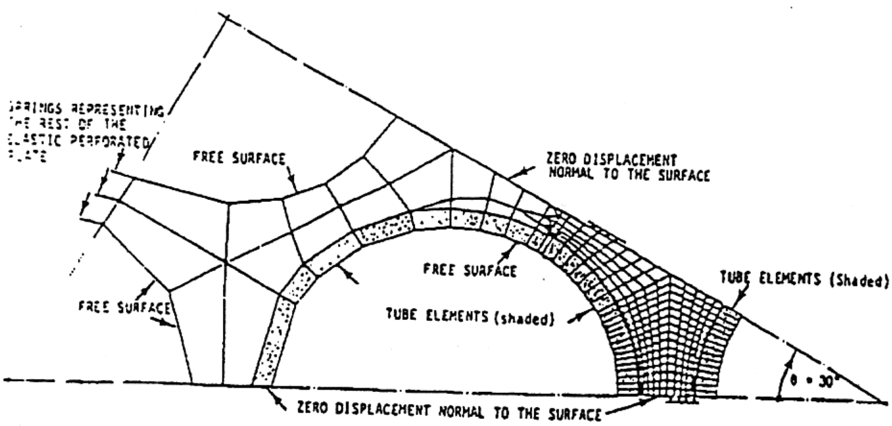


Figure 6 Finite Element Model With Tubes In Neighboring Holes

MODEL WITH TUBES PREVIOUSLY EXPANDED IN NEIGHBORING HOLES

The model used to investigate the effect of neighboring tubes which had been expanded prior to the tube being analyzed is shown in Figure 6. The model of the central tube and the analyzed segment of the tubesheet, as well as the boundary conditions, are identical to that shown in Figure 5. Additional elements simulating response of the tube are added in the six holes directly surrounding the hole where the analyzed tube is being expanded.

ANALYSIS OF AXIAL EXTENSION OF TUBES

Very high radial pressures imposed on the tube during expansion result in a three-dimensional state o strain. Expansion permanently increases the diameter of the tube. The required constancy of material volum also requires the tube to thin when the material flows axially during rolling. Thinning of the tube is often used as a practical measure for controlling the degree to which the tube is expanded.

The finite element model used to analyze the three-dimensional plastic flow of the tube material is shown in Figure 7.

The model is axisymmetric and the thickness of th heavy ring is selected to simulate best the average response of the ligaments surrounding the expanded tube The thickness of the ring corresponds to the thickness of the perforated plate. Only 50 mm along the length of the tube is expanded by imposin radial expansion. The model contains 303 four-node axisymmetric elements and 65 two-dimensional interface (gap) elements.

ANALYSIS RESULTS

The characteristic developmen of the plastic zone in ligaments o expanded tubesheet is shown in Figure 8. Yielding begins (line 1 at the location where the radial ligament stiffens the hole boundary After some growth of the plastic area at this location (2), yielding occurs at the hole boundary (3) on the other side of ligament. Continued enlargement of the hole finally causes cross yielding of the ligament. Plastic hinges occur away from the minimum ligament section. The central portion of the ligament between the hinges remains rigid and can be forced out radially causing excessive plastic straining at hinge locations, if the tube expansion is continued too far.

The residual contact loads after unloading are shown by the dotted line in Figure 9 as a function of radial expansion of the tube. Also shown is the corresponding thinning of the wall. Note two coordinate systems on the left side of the graph. They indicate thinning of the tube measured from the instant where the expander first comes in contact with the tube and thinning measured from the point where the expanded tube touches the hole surface.

Note that differences between the yield strength of the tube and tubesheet materials affect stresses significantly. Tubes which have higher yield strength spring back elastically further than the tubesheet. Due to this effect, gaps between the tube and hole surface tend to open during unloading when the expansion of the tube is not sufficient. Gaps opposite to radial ligaments between the neighboring holes close within a reasonable range of straining. However, along the tangent ligaments, which are less stiff in the radial

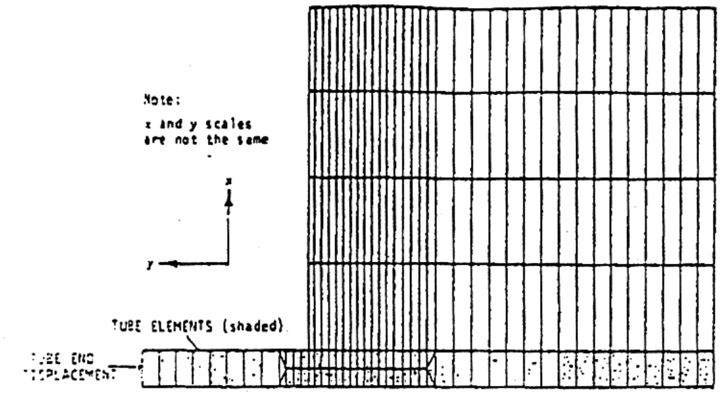


Figure 7 Axisymmetric Finite Element Model for Analysis Of Axial Extension of Tube

CURVE NO.	RADIAL EXP. in. x 10³	RADIAL EXPANSION ÷ RADIAL EXPANSION @ INCIPIENT YIELDING
1	12.010	1.0000
2	12.027	1.0014
3	12.1325	1.0102
4	12.449	1.0365
5	12.871	1.0717

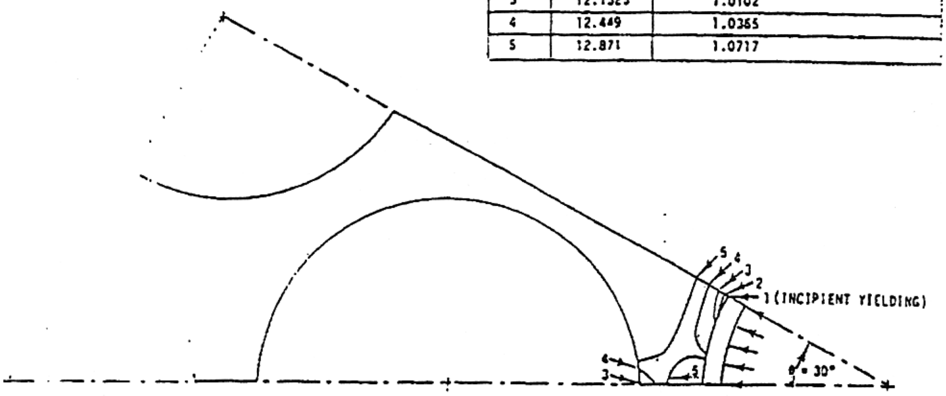


Figure 8 Progression Of The Plastic Zone In The Tubesheet

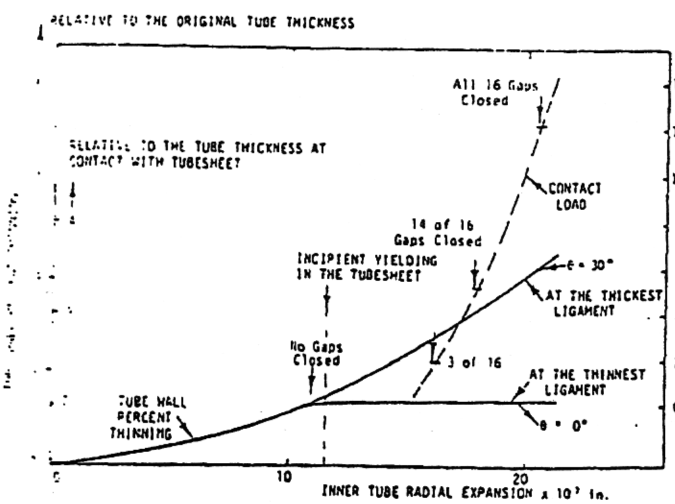


Figure 9 Thinning Of The Tube And Residual Contact Load For Expansion Of First Tube

direction, much larger straining is required for gaps to close. This is illustrated in Figure 9.

The response of the joint with tubes in neigl boring holes is shown in Figure 10. The residual loads are not essentially affected by the presenc of the tubes. The gaps, however, remain closed wi less straining than in the case of the single tube expansion shown in Figure 9.

The stress distribution during the expansion process is illustrated by the stress intensity cor tour plots given in Figures 11 and 12 which show t isostress line distributions for two degrees of expansion. Radial expansion for the case shown ir Figure 11 is defined in Figure 8 by the line 2. Correspondingly, the expansion in Figure 12 is defined by line 5.

Figure 13 illustrates the stress distribution for the model with neighboring tubes which can be compared with Figure 12. There are no significant changes in stress pattern due t the presence of tubes in neighb ing holes.

Axial displacement and thinning of the tubes are given Figure 14. Thinning varies with axial location. Due to Poisson' effect, the expanded tube contracts axially during elastic deformation. Continued plastic straining, however, results in gradually increasing radial compression of tube material. Modified configuration of the stress tensor finally reverses deformation causing the tube to extend its length. No essential net extension of the tube occurs within the reasonable range of expansion.

CONCLUSIONS AND RECOMMENDATIONS

Expanding of the tubes shoul ensure sufficient residual contac force so that the resulting friction can support axial load in the tubes.* It is alsc desirable that the clearance between the tube and the hole surface is eliminated such that no crevice opens after expansion.

** Expansion process for closely spaced tubes has to be very carefully controlled in order to prevent overstraining of thin ligaments. Low ligament efficiency and higher yield strength of the tubes than yield strength of perforated material reduce effectiveness of the tube expansion.

For the particular ligament efficiency analyzed herein, the expansion of the tube should be performed such that thinning of the tubes measured from the instant of first contact of the tube with the hole surface should be between 2% and 3%.

REFERENCES

NADAI, A., Theory of Flow and Fracture of Solids, McGraw-Hill Book Company, Inc. 1950

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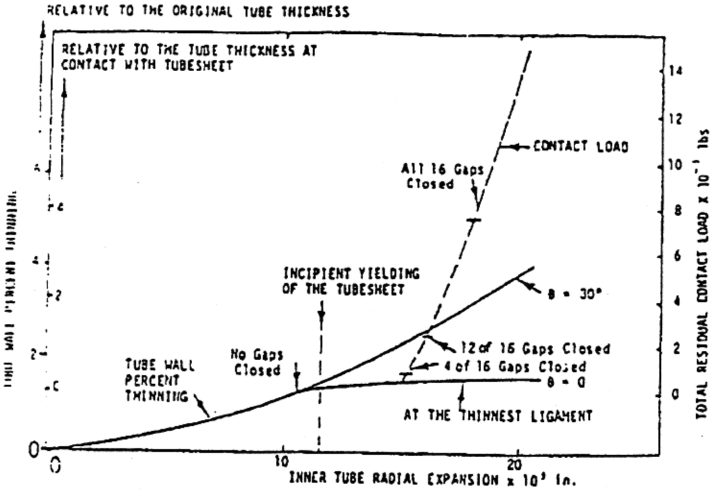

Figure 10 Thinning of The Tube and Residual Contact Loads For Tube Expanded In The Tubesheet Strengthened By Neighboring Tubes

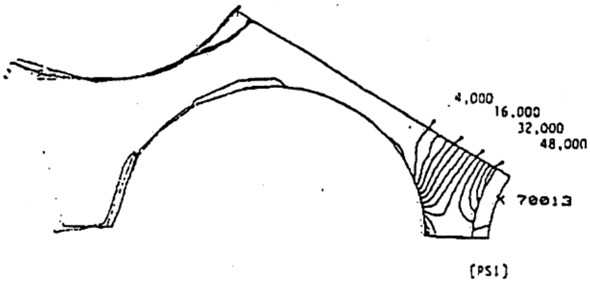

Figure 11 Stress Intensity Field For Expansion Of A Single Tube With Radial Expansion Corresponding to Line 2 in Figure 8

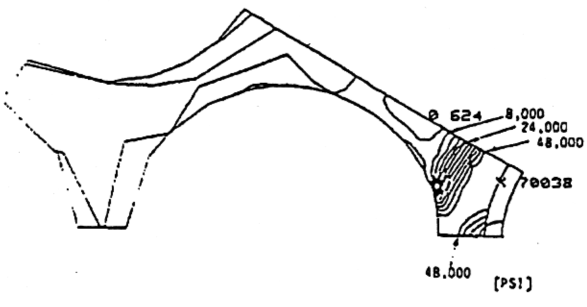

Figure 12 Stress Intensity Field for Expansion of Single Tube With Radial Expansion Corresponding to Line 5 in Figure 8

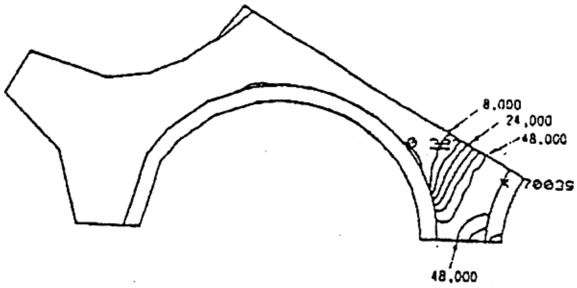

Figure 13 Stress Intensity Field For Tube Expande in Tubesheet Strengthened by Neighboring Tubes With Radial Expansion Corresponding to Line 5 in Figure 8

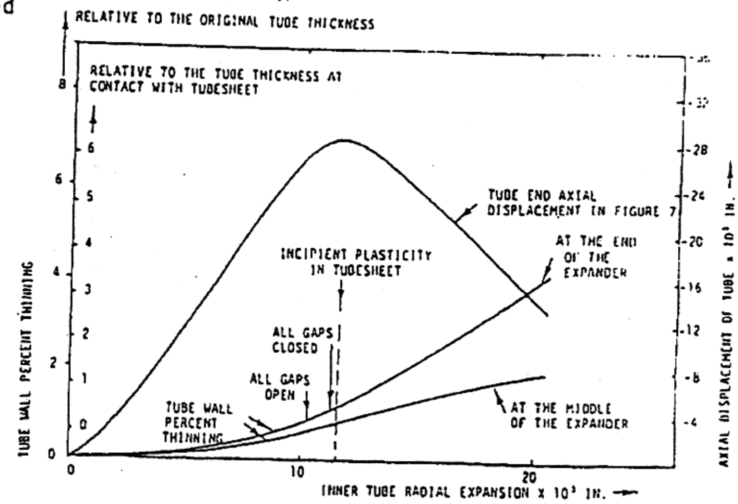

Figure 14 Thinning and Axial Displacement of The Tube