The Holding Power and Hydraulic Tightness of Expanded Tube Joints: Analysis of the Stress and Deformation

By J. N. GOODIER[1] and G. J. SCHOESSOW[2]

The expanding process used for making the tubes of boilers tight and in other applications is idealized to form a problem in the theory of plasticity. This problem is solved in order to find out how far the factors taken into account in this theory are adequate to explain the results obtained in tests, mainly those reported in the companion paper of Grimison and Lee (19).[3] The pressure left between the tube and plate or seat, which gives it tightness and contributes to its strength, is the principal object of calculation. Its variations with the yield stresses of tube and plate material, with degree of expanding, and with the thickness of the plate, are obtained in graphical form. Numerical comparison with tests is made for six joints, with fair agreement in five, the theoretical values being within 12 per cent. All are on the low side, and this is to be expected from the fact that the theory disregards strain-hardening, stress in the direction of the tube axis, and possible differences between expanding by a three-roller tool and expanding by uniform pressure within the tube.

Introduction

If the end of a tube is inserted loosely into a hole in a plate (Fig. 1) and is then plastically expanded by internal pressure, removal of the pressure leaves the tube bearing tightly against the plate, and the result is called an expanded tube joint. This process is used in the construction of water-tube boilers. The joint must not only be tight against leakage but must have structural strength, to contribute to the support of dead load and to resist the stresses and strains of temperature changes. The tightness against leakage will evidently depend upon the surface conditions of the tube end and the plate hole, the uniformity of the expanding process, and the residual radial pressure left between tube and plate. The structural resistance to tension, torsion, and bending depends upon the friction in the joint. This in turn depends upon the surfaces and upon the residual radial pressure. The discussion of this pressure is the principal object of the present paper. One form of expanding tool, in which three rolls are pressed against the inner tube surface and travel around it, is illustrated in Fig. 2. Part 5 is a flaring roll to turn over the end of the tube.

It will be seen that the expanded joint resembles the shrink fit, or rather the expansion fit, where the slightly oversize tube would be chilled, inserted in the hole, and allowed to expand against the plate by warming up. In the expanded joint, the "oversize" of the tube is produced by the internal forces after the tube has been inserted. Whereas the shrink or expansion fit is normally such

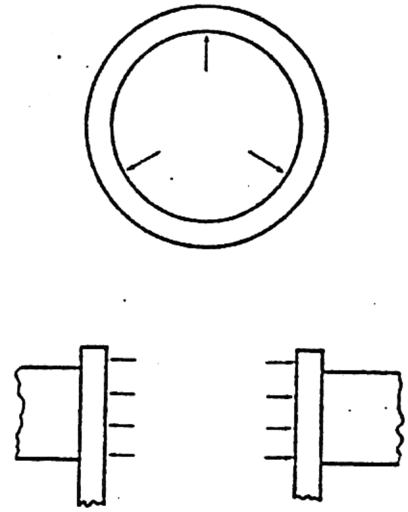

Fig. 1 Expanding Tube Plastically by Internal Pressure

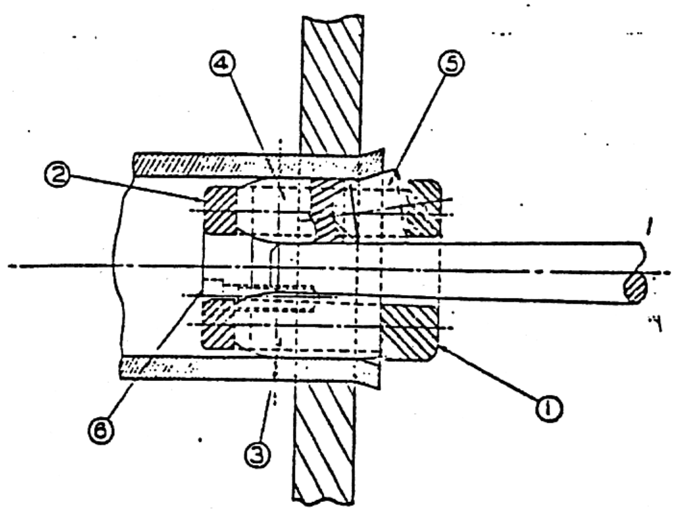

Fig. 2 Three-Roller Expanding Tool

that only elastic stresses are produced, and the oversize of the tube is under immediate control, the expanded joint on the other hand necessitates plastic stresses, at least in the tube, and the effective "oversize" produced is not under immediate control.

In seeking to understand how an effective oversize of the tube is created by the expanding process, it may be helpful to consider a strip of tube material confined in a perfectly rigid frame, Fig. 3(*a*), and subjected to the action of a single roll. The material under the roll, which is yielding plastically, tends to extend

[1] Consulting Engineer, The Babcock & Wilcox Company; Professor of Engineering Mechanics, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

[2] Engineer, The Babcock & Wilcox Company, Barberton, Ohio. Mem. A.S.M.E.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

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longitudinally and laterally. It is able to extend somewhat longitudinally by putting the remainder of the strip under elastic compression. If the roller is now removed, some of this compressive stress remains, the element under the roll retaining some permanent set. The rolling process may be imagined as the repetition of this process on the different elements.

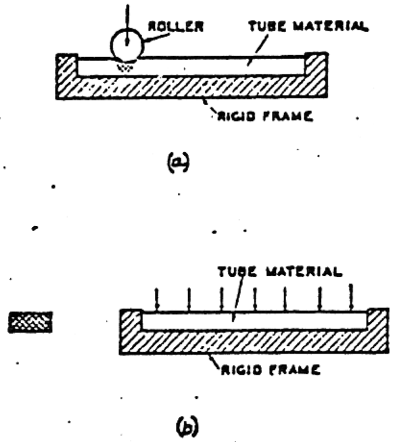


Fig. 3 Diagram Showing How Effective Oversize of Tube Is Created by Expanding

(*a*, Model to illustrate roll expanding; *b*, model to illustrate uniform-pressure expanding.)

When the strip, instead of being straight, is curved up into a circle, retaining its rigid backing, we have the case of a tube in a rigid plate. The residual longitudinal stress becomes a residual circumferential stress requiring a residual radial pressure for equilibrium.

If, instead of the roll, we have a uniform pressure, Fig. 3(*b*), only lateral, not longitudinal, extension is possible, and this condition is unfavorable for the development of a longitudinal residual stress. However, when the pressure is applied within a circular hole in a plate, it is possible to have permanent set created in a ring zone surrounding the hole. If the whole plate is elastic under the pressure, the radial and circumferential strains would be equal in magnitude but opposite in sign at any point, and both decrease outward. If the pressure is increased, these strains near the hole begin to exceed those elastically possible, and in the zone where this occurs there will be permanent set.

In the actual rolling, the pressure on the rolls gradually increases and the rolls make several circuits. From the first therefore an all-around pressure between tube and plate is being gradually built up, in virtue of the all-round permanent expansion of the tube already established. At any stage there is combined with this the local stresses created by the rolls wherever they happen to be. These localized stresses are absent in uniform-pressure expanding. The differences are well illustrated by Figs. 4(*a*), (*b*) from a paper by Thum and Jantscha (10), showing the patterns of slip lines in the two cases. In Fig. 4(*b*), owing to uniform pressure, only the well-known spiral system of slip lines appears. In Fig. 4(*a*), due to a roll expander, similar slip lines appear but accompanied by a narrow inner blackened zone, which is evidently a ring succession of zones of localized contact stress under a roll—the plastic zone of Fig. 3(*a*) repeated all around the ring.

Analysis of Stress During and After Expanding

Some knowledge of the state of stress both during and after expanding is essential not only for an understanding of the expanding process but also for the consideration of the plate under service stresses. The favorable circumstance that the strains created in expanding are not or rather do not need to be large, makes the problem an appropriate one for analysis by the mathematical theory of plasticity, ignoring rise of stress beyond the yield point. Typical physical characteristics of tube and plate materials and measurements of the strains are given in the paper of Grimison and Lee (10).

Only the results are given in this section. Derivations will be found in the Appendix.

It is necessary for reasons of mathematical difficulty to restrict the calculations to joints made by uniform internal pressure, as in some of the tests of Jantscha (11). Jantscha found, however, that the joints thus made were as good as his rolled joints. The differences between the stresses produced by the two methods have already been discussed qualitatively.

The law of plastic flow adopted is that of constant strain energy of shear (von Mises-Hencky hypothesis). The stress parallel to the axis of the tube is taken as zero (plane stress). The other extreme case, that of zero axial strain (plane strain), was also investigated, but the stress curves were not much different.[4]

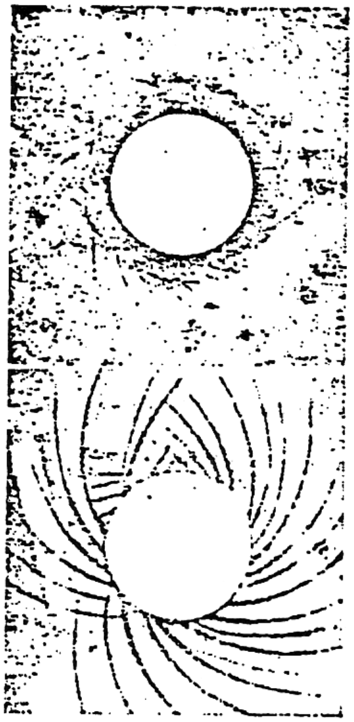


Fig. 4 Patterns of Slip Lines in Expanded Tubes

[*a* (*upper*) Caused by roller expander; *b* (*lower*) caused by uniform pressure.]

Conditions are taken as uniform through the plate thickness.

We consider first the simplest case, where there is no clearance and tube and plate are of the same material. They can then be treated as one piece of metal, i.e., an infinite plate with a hole of diameter equal to the inner tube diameter. When pressure is applied to the inside of this hole, the stress distribution is at first elastic, but plastic flow begins at the hole, when the pressure reaches $s_0/\sqrt{3}$, where s_0 is the yield stress in simple tension. As the pressure is increased beyond this, the plastic zone spreads out from the hole, but under the assumption of plane stress, this does not continue indefinitely. When the radius of the plastic zone has reached 1.75 times the radius of the hole, and the pressure the

[4] Plane strain, however, does not set a limit to the pressure attainable in the hole, whereas in plane stress, this limit governs the maximum residual pressure.

sponding values $2s_0/\sqrt{3}$ or 1.15 s_0, both cease to increase. ıy attempt to increase the pressure further is defeated, in the ealized material, by axial extrusion. It seems entirely probable at a similar situation exists for the actual material.

Records of tests by The Babcock & Wilcox Company on 3¹/₄-in- D tubes of thicknesses 0.17, 0.29, 0.43, 0.50 in. show that the ll scale on the plates popped to distances of 0.6, 0.6, 0.5, 0.6 in. ıe diameters of the plastic zone thus estimated to the inner ımeter of the tube are 1.53, 1.67, 1.78, and 1.98 as compared th the theoretical limit 1.75, ignoring clearance. In another se, thickness 0.17 in., hardness measurements in the plate as ell as scale-popping observations gave a value 1.65.

Returning to the theoretical problem, the curves of the radial ress s_r and the circumferential stress s_t, due to the limiting pres- re can be calculated. They are shown in Figs. 5 and 6. In g. 5, the stress beyond the limit of the plastic zone is of elastic rm, although of course it is not the elastic stress corresponding rectly to the pressure in the hole. The broken line shows how it ould continue into the plastic zone. The actual plastic stress s_r the full curve below it and is practically a straight line.

In Fig. 6, the curve BJ shows the stress s_t beyond the plastic ne, continuing as BA within the plastic zone. It will be seen at, even during the application of the pressure, the circumfer- tial stress is compressive in a ring of thickness 0.20 a, where a the radius of the hole.

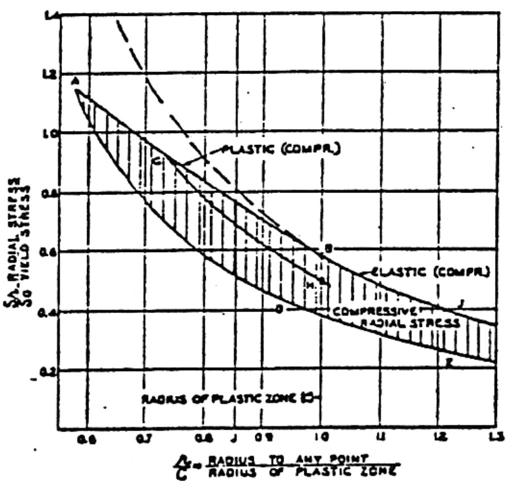


Fig. 5 Radial Stress Curves

In order to determine the residual stress remaining after the essure in the hole has been removed, it is assumed that the anges of stress and strain during decrease of pressure are purely istic. This is known to be characteristic of unloading in simple nsion, compression, and torsion (17), and the assumption that is also valid for compound stress is invariably made in the lculation of residual stress (18). We may then regard unload- g as the superposition of an equal radial tensile stress in the ıle, producing the elastic curves ADE in Fig. 5, FDE in Fig. 6. ıese are tensile and compressive stresses, respectively. They e, however, inverted in order to exhibit what stresses remain ıen they are superposed on the preceding stresses. The re- ainders, shown by the intercepts in the shaded areas, are the sidual stresses. The maximum residual radial stress is 0.215 s_0 . occurs at a radius approximately 1.45 a. The circumferential ress s_t is compressive up to 1.58 a with a maximum value 1.73 s_0 . the hole. This value, however, exceeds the plastic limit of 1.15 which governs both principal stresses.[5] The values of the sidual s_t for radii up to about 1.1 a also exceed this limit.

[5] See Appendix

It would appear therefore that the process of unloading cannot be a purely linear one as we have supposed. As it proceeds, it continually adds, near the hole, compressive circumferential stress to a stress of the same nature which was induced on loading. Elastic recovery may occur until the circumferential stress reaches 1.15 s_0, but after that, the recovery becomes inelastic. No attempt is made in this paper to take account of such an inelastic unloading process, and the residual circumferential stress of 1.73 s_0 remains as a defect in the theory. In spite of this, reasonably good agreement with test results is found.

To find the residual pressure between tube and plate, we have merely to take the intercept in Fig. 5 which occurs at an abscissa corresponding to the outside of the tube. If the outer radius is b the required abscissa is given by the value of b/c ($c = 1.75\,a$). We are assuming here that the expanding pressure is always the

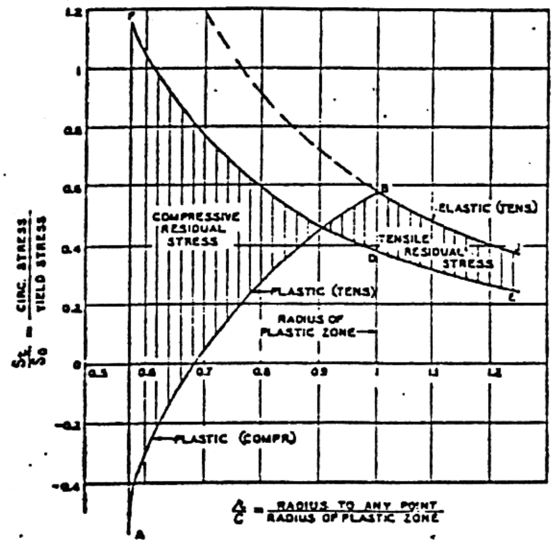


Fig. 6 Circumferential Stress Curves

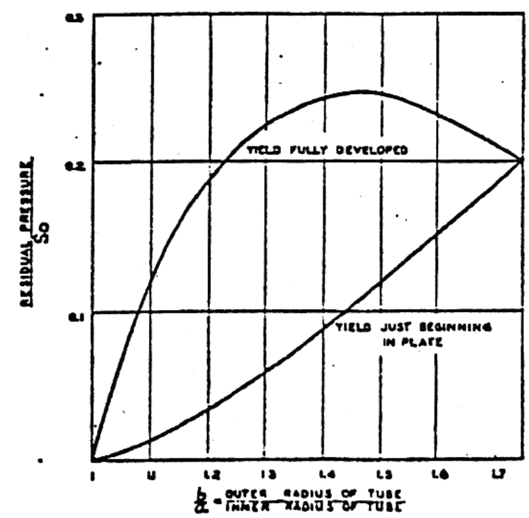


Fig. 7 Variation of Residual Pressure With Thickness of Tube

limiting pressure 1.15 s_0. It is thus possible to determine the residual pressures for a range of values of b/a. Such values are plotted in Fig. 7 (upper curve).

To examine the commonly held view that the best joint is obtained when the plate is stressed only up to its elastic limit, but not beyond, we may start from the point B in Fig. 5, which represents the limiting elastic radial stress. Whatever the tube thickness, the plastic curve will proceed from B as before. We may, however, stop at any point G and take that as corresponding to the inner tube radius. Then the ordinate to G represents the highest pressure to be used in expanding if the elastic limit of the plate is not to be exceeded. The elastic unloading curve GH must then be drawn from G, and the intercept HB now gives the residual pressure. Again, this can be determined for various ratios of tube outer and inner radii. These values also are plotted in Fig. 7 (lower curve). They give much lower values than those obtained previously by allowing the pressure in the hole to reach the plastic limiting pressure, and the plastic zone to extend into the plate. Thus the view that the plastic zone, with its own inner zone of residual compressive circumferential stress, acts to shield the tube from the elastic reaction of the plate, does not justify the conclusion that the plastic zone should not extend into the plate. The shielding effect is present, but by not stopping when it begins we are able to go to a higher expanding pressure which more than offsets the shielding effect by producing a general increase in stress.

The implications of the theory as to overexpanding, i.e., the decrease of residual pressure and holding power beyond an optimum degree of expanding, may also be examined with the help of Fig. 5. The point B in all cases represents the boundary of the plastic zone. The corresponding radius c is now taken to be within the plate. We can choose any point G on the plastic part of the stress curve and take it as corresponding to the inner radius of the hole. We can thus obtain any ratio c/a, within limits, corresponding to some degree of expanding (radius of the plastic zone) and expanding pressure, less than the possible maxima. We may then take an abscissa J between G and B, to represent the outer radius of the tube, and read off the corresponding intercept at J between the curves GB and GH. The values thus obtained are plotted in Fig. 8. Each curve shows the variation of residual pressure with the extent c of yield in the plate, for a definite ratio b/a of the two tube radii. It will be seen that in all cases the residual pressure increases with c.

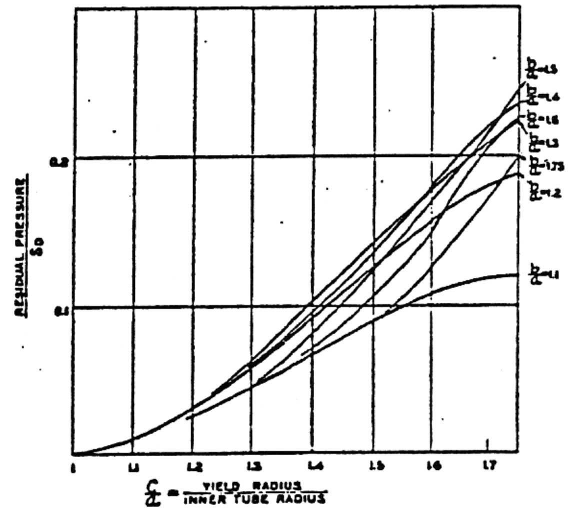


FIG. 8 VARIATION OF RESIDUAL PRESSURE WITH RADIUS OF PLASTIC ZONE

Thus the theory, as far as it has been carried, yields no clue to the explanation of overexpanding. This negative result, however, serves to direct our attention to the factors which the theory has not taken into account. Of these the most suggestive is the thinning of the tube wall during expanding. If we treat this as a simple reduction of b/a, we conclude from the upper curve in Fig. 7 that this will explain some reduction of residual pressure, but only if b/a is less than about 1.4. For instance, in the case of a $3^1/_4$-in-OD tube, of 0.32-in. wall thickness, $b/a = 1.24$, and if the extrusion and, consequently, wall thinning is 15 per cent, the value of b/a after expanding is about 1.21. The reduction of residual pressure, from Fig. 7, is from 0.21 s_0 to 0.19 s_0, or 10 per cent.

The initial clearance between tube and plate has so far been ignored. It makes no difference, however, to the foregoing results, as far as the theory is concerned. When the expanding begins, the tube expands with a plastic stress distribution (the lowest curve in Fig. 9) different from that of Fig. 5. When contact is established, the pressure on the outer tube surface grows, and the stress distribution progressively changes. But the curves cannot rise beyond the curve of Fig. 5, and this is the final curve just as without clearance. The tests of Oppenheimer, however, showed a marked effect of clearance (15).

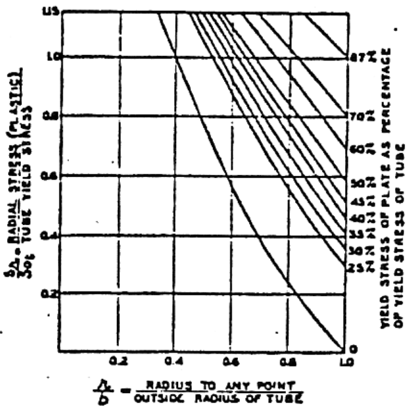


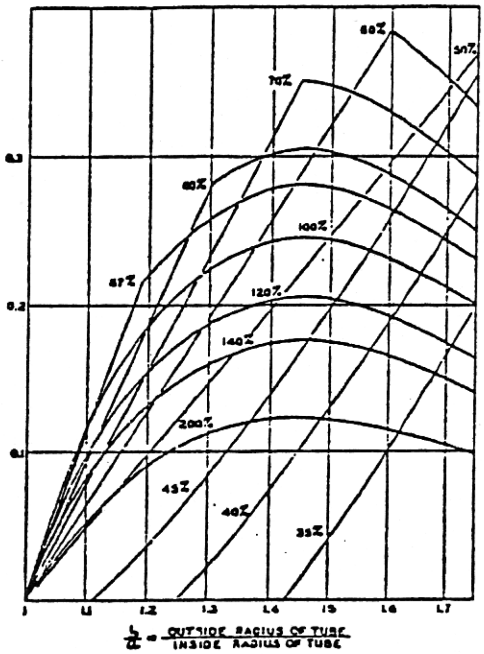
FIG. 9 VARIATION OF RADIAL STRESS OVER THICKNESS OF TUBE FOR LIMITING INTERNAL PRESSURE, VARIOUS EXTERNAL PRESSURES

In actual joints the yield stresses of the tube and plate materials are usually different. The effect of this will therefore be considered before numerical comparisons are made.

EFFECT OF DIFFERENT YIELD POINTS OF TUBE AND PLATE

Plate Yield Point Higher Than That of Tube. In the case already considered, of identical yield points of plate and tube, the best joint was obtained when the limiting pressure 1.15 s_0 was reached inside the tube. Since the radial stress due to the applied pressure invariably decreases along the outgoing radius, the radial stress in the plate never reaches the limiting value 1.15 s_0, except for a tube of infinitesimal thickness. For a tube of any given thickness, there is a radial pressure, defined by the appropriate point on the plastic curve AGB of Fig. 5, which is required on the outer tube surface. A plate of higher yield point is able to supply this pressure equally well. The stress distribution within the plate will be different. It becomes wholly elastic if the yield point of the plate is twice that of tube. But the plastic curve within the tube material is still that of Fig. 5, and the residual pressure is still found by the same construction—the intercept between the plastic curve AGB and the elastic curve ADE at the value of r/c which corresponds to the outside of the tube, that is, b/c. The s_0, however, which figures in the dimensionless ordinates, is of course that of the tube, not that of the plate. The two will be distinguished by the subscripts t and p, as s_{0t} and s_{0p}. The residual pressures for various values of b/a are therefore given by the upper curve of Fig. 7, the ordinates being values of residual pressure divided by s_{0t}.

We have the conclusion then that no advantage is gained by using a harder plate, as compared with a plate as hard as the tube, and this result is incorporated in Fig. 10. There, however, the ordinates are residual pressures divided by s_{0p}. Since we are considering increases of s_{0p} the ordinates of the curve of Fig. 7



10 RESIDUAL PRESSURES FOR VARIOUS TUBE PLATE HARDNESSES AND VARIOUS THICKNESSES

: be reduced in proportion. Thus we obtain the curves :d 120, 140, 200 per cent. The curve marked 100 per cent - same as that of Fig. 7. The percentages denote the ratio ate yield point to tube yield point.

late Yield Point Lower Than That of Tube. It is necessary as eliminary to consider the plastic stress in a tube when the nal pressure has the limiting value 1.15 s_{0t}, but the pressure he outside of the tube has any value. This problem offers no culty and is solved in Nádai's "Plasticity"(18).[6] The solution presented by the curves of Fig. 9. Starting at the right-hand of a curve, we begin with the prescribed outside pressure. plasticity law and the conditions of equilibrium then deter- the curve completely, and it terminates at a pressure of 1.15 and at a radius depending upon the outside pressure chosen. re is thus a lower limit to the inside radius. Any thicker tube be in plastic equilibrium only for a lower external pressure. lowest curve, for zero external pressure, has its inner ter- us at $r/b = 0.335$. Thus if b/a exceeds 1/0.335, or 2.98, plastic equilibrium throughout the tube is not possible. The will be elastic in an outside zone.

onsider now, as an example, a plate whose yield point is 60 cent of the tube yield point or 0.6 s_{0t}. Its limiting radial sure is then 60 per cent of 1.15 s_{0t}, and this ordinate is marked er cent on the right of Fig. 9. We may then follow the plastic curve which passes through this point and proceed inward value a/b of r/b corresponding to the inside of the tube. stress reached on the curve is the highest internal pressure ch can be applied without exceeding the limit set by the plate. m this point on the curve the elastic curve can be drawn, and residual pressure read off as the intercept between it and the ic curve at $r/b = 1$. This construction can be carried out fferent values of a/b until the inner terminus of the curve is hed. Results are plotted as the straight-line part of curve ked 60 per cent in Fig. 10, which stops at $b/a = 1.6$, or $a/b =$ 5, the abscissa of the inner end of the 60 per cent curve in Fig. 9. or thicker tubes, the limiting stress of the 60 per cent plate is reached, and we have to go to a lower curve, accepting a lower stress in the plate. We can retain the limiting stress 1.15 s_{0t} at the inside of the tube. The situation is thus no longer governed by the limiting stress of the plate, but by that of the tube. There is consequently a change in the nature of the curve for large b/a, as appears in Fig. 10. The curve in fact continues in the same way as Fig. 7.

The curves marked 50, 70, 80, and 87 per cent in Fig. 10 are determined in the same way. The same construction is used for the curves marked 45, 40, and 35 per cent. These, however, show here residual pressure below certain values of b/a. This feature begins at 50 per cent. An elastic curve (for the infinite plate) drawn through the right-hand point of the curve marked 50 per cent in Fig. 9 has, at that point, its tangent in common with the plastic curve, and, proceeding to the left, the elastic curve lies above the plastic curve. For the lower plastic curves, however, the elastic curves drawn through their right-hand ends lie, for some distance to the left, below the plastic curves. If we take a point on one of these plastic curves, within this distance, to correspond to the inner radius of the tube, the elastic curve drawn from this point will lie above the plastic curve at $r/b = 1$. The intercept will then give a negative residual pressure, a tension, which would exist if the tube adhered to the plate. It would thus indicate zero pressure when there is no adhesion. Thus according to the present theory, no joint can be made if the tube is much harder than the plate, unless the tube is sufficiently thick.

Fig. 10 gives a complete account of the residual pressures obtainable for the range 35 per cent to 200 per cent of s_{0p}/s_{0t} and the range 1 to 1.75 of b/a.

In order to exhibit more clearly the effect of varying the yield point of the plate, values from these curves are plotted in Fig. 11

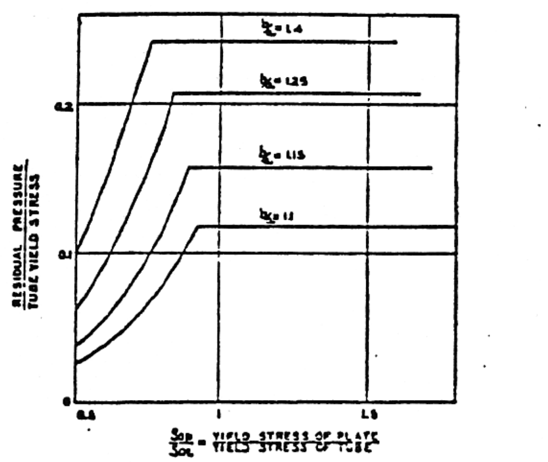


FIG. 11 EFFECT OF HARDENING PLATE

in a different manner. The ordinates, residual pressure divided by s_{0t}, represent directly the residual pressure if s_{0t} is kept constant. The abscissas, s_{0p}/s_{0t}, correspond to changing s_{0p} under the same condition. We have then one curve for each thickness of tube. It will be seen that increasing the yield point of the plate has a very favorable effect so long as s_{0p}/s_{0t} is below a certain limit depending upon b/a. It is, for instance, 0.83 for $b/a = 1.25$. But any further increase in the yield point of the plate is without effect on the residual pressure, although it will mean a somewhat different stress distribution in the plate. The reason for this is of course that above this limit, the situation is governed by the limiting stress which the tube material can bear, and the plate is no longer called on for its limiting stress. The conclusion may be expressed by saying that if the yield point of the plate is substantially less than that of the tube, better results may be expected by using a harder plate. This state of affairs is the usual ... Quantitative comparisons are made in the text following.

[6] Equation (30) on p. 191 contains an error. The corrected form

The results recorded in Fig. 10 are again replotted in Fig. 12, to show the effect of changing the yield point of the tube when that of the plate is kept fixed. The ordinates then correspond to the residual pressure, and the abscissas to the yield point of the tube, and there is a curve for each value of b/a. The linear left-hand parts, corresponding to the softer tubes, are governed by the limiting stress of the tube material. The curved parts beyond

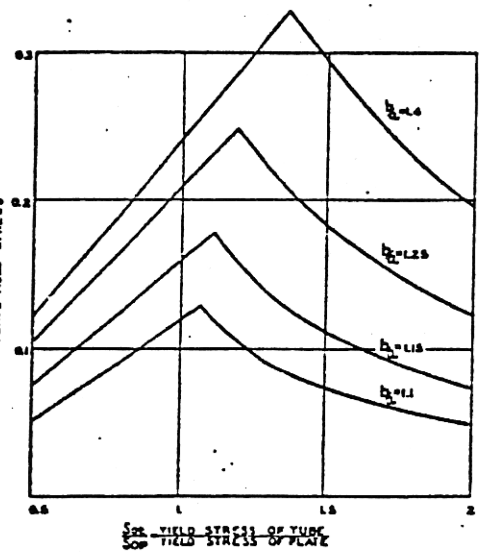


FIG. 12 EFFECT OF HARDENING TUBE

the breaks are governed by the limiting stress of the plate material, which there does not permit that of the tube material to be reached. It will be seen that if the tube is substantially softer than the plate, the residual pressure can be raised by using a harder tube. But the use of a tube substantially harder than the plate may result in a smaller pressure.

Tests have been made by The Babcock & Wilcox Company on the holding power of soft tubes rolled in hard plates, and hard tubes in soft plates. The details and results are as follows:

Tube hardness	100 Bhn	320 Bhn
Plate hardness	302 Bhn	107 Bhn
Initial slip	18500 lb	11500 lb
Ultimate load	79000 lb	36000 lb

Tube, 3¼ in. OD; wall thickness 0.22 in.

No values of the yield points are available, but the hard plate would without doubt correspond to a point on the straight part of the appropriate curve in Fig. 11 ($b/a = 1.15$). The residual pressure would then be $0.158\ s_{0t}$. When the hard material is used for the tube, we would have to conclude from Fig. 10 that no joint is possible if s_{0p}/s_{0t} is less than about 0.45. The theoretical curves thus suggest that the hard plate – soft tube combination is very much better than the reverse, and this is in qualitative agreement with the results for the holding power.

If we consider two materials, one having twice the yield point of the other, with $b/a = 1.15$, the hard plate – soft tube combination gives a residual pressure of $0.158\ s_{0t}$ or $0.158\ s_0$ if s_0 is the yield point of the softer material. The hard tube - soft plate combination gives $0.074\ s_0$. The advantage is about 2 to 1 in favor of the hard plate – soft tube combination.

COMPARISON WITH TEST RESULTS

Table 1 shows the residual pressures obtained by the preceding theory for joints made and tested by Grimison and Lee (omitting one of alloy tubing showing no yield point). To illustrate the determination of the theoretical value, consider Case 2. The yield stress of the plate material, from test, is 31,500 psi, that of

TABLE 1 COMPARISON OF THEORETICAL RESULTS WITH MEASURED PRESSURES ON 3¼-IN-OD TUBES IN PLATES 1½ IN. THICK

Case no.	Tube-wall thickness, in.	b/a	Plate yield stress s_{0p}, psi	Tube yield stress s_{0t}, psi	s_{0p}/s_{0t}	Greatest measured residual pressure, psi	Calculated residual pressure, psi
1	0.17	1.12	31500	55000	0.59	5600	2400
2	0.32	1.25	31500	40000	0.79	8100	7200
3	0.32	1.25	26500	35000	0.76	6400	5720
4	0.50	1.44	31500	37800	0.83	10100	9200
5	0.32	1.25	very high	40000	exceeds 0.83	8400	8160

the tube material 40,000 psi, so that $s_{0p}/s_{0t} = 0.79$. The value of b/a is 1.25. Using the corresponding curve of Fig. 11, we find that the ordinate for the abscissa 0.79 is 0.18, so that the residual pressure is $0.18 \times 40{,}000$ or 7200 psi.

With the exception of Case 1, the agreement with the test values is satisfactory. The theoretical values are all on the low side. This may be attributed to three causes, i.e., the neglect of the rise of the stress-strain curves beyond the yield point; differences between roll- and uniform-pressure expanding; and the assumption of plane stress. As to the latter, Grimison and Lee (19) find evidence that the axial frictional stress between tube and plate increases the residual pressure. Such stress is taken as zero in plane stress. There appears to be no evident reason for the large discrepancy of Case 1.

Prerolled Plates. The fact that in normal practice the tube has a considerably higher yield point than the plate (e.g., 35,000 psi as compared with 26,500), a difference which is increased by cold-working, leaves room for improvement by the use of a harder plate. The original plate can, however, be conveniently hardened, in the zone around the hole where hardness is required, by expanding the undersized hole in the plate by means of an ordinary expanding tool. This idea has been tested by The Babcock & Wilcox Company and, in the results available to date,

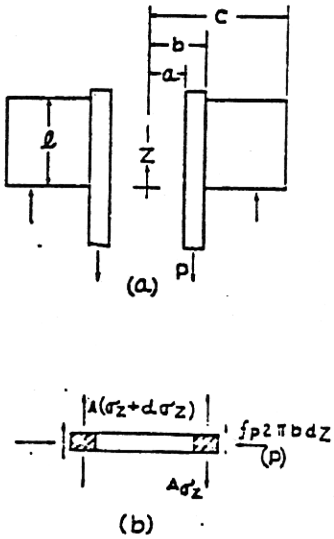


FIG. 13 EXPANDED JOINT UNDER PULL-OUT FORCE

a maximum residual pressure of 8290 psi was found for 3¼-in-OD tubes 0.32 in. thick. The maximum found for ordinary plates was 6400 psi, this being Case 3 of Table 1. The appropriate curve in Fig. 11, that for $b/a = 1.25$, shows that the use of plates such that $s_{0p}/s_{0t} > 0.83$, which correspond to the flat part of the curve, can raise the pressure to $0.208\ s_{0t}$, or, since $s_{0t} =$ 35,000, to 7300. This falls short of the test value 8290 by 12 per cent, the same percentage defect as in Case 3 itself. It thus seems very probable that all the improvement available by using harder plates can be secured by prerolling.

The initial linear form of the curves of Fig. 10 for the lower ues of b/a is in agreement with conclusions from tests by Jant- (11) and Ries (13).

Calculation of Frictional Holding Power

To calculate the frictional holding power the tube is treated as ted into a plate, with an external radius c and hole radius b, ith a uniform pressure p_0 at the common surface. The outer ameter is regarded as finite to correspond with test specimens. ension being applied to one end of this cylinder will tend to duce its diameter and so partially relax, in an axially non-iform manner, the original fit pressure p_0. Let the actual essure between tube and plate be p, a function of z (Fig. 13), hen a pull P is applied to the tube. The corresponding axial ress in the tube σ_z, also a function of z, is taken as uniform over e thickness of the tube. Vertical equilibrium of the element the tube shown in Fig. 13(b) gives

$$A\frac{d\sigma_z}{dz} + 2\pi bfp = 0 \qquad [1]$$

here $A = \pi(b^2 - a^2)$, the cross-sectional area of the tube, and f the coefficient of friction.

On application of the pull-out force P, the pressure p_0 is reduced the height z by $p_0 - p$. This permits a decrease of the radius the hole in the plate by the amount

$$\Delta b_p = (p_0 - p)\,\frac{b}{E}\left(\frac{c^2 + b^2}{c^2 - b^2} + \nu\right) \qquad [2]$$

here E is Young's modulus and ν Poisson's ratio. This formula taken from the thick-cylinder theory, for conditions uniform ng the axis. It can also be established for linear variation of ss along the axis, provided certain shearing stresses on the be ends are accepted. The variations ultimately found in the esent case are nearly linear.

The reduction of the pressure by $(p_0 - p)$ permits an increase external radius of the tube of

$$(p_0 - p)\,\frac{b}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu\right) \qquad [3]$$

it the tensile stress σ_z involves a decrease of $\nu\sigma_z b/E$. Equating e net decrease to the decrease of radius of the plate hole Equa-on [2], we obtained an equation which reduces to

$$\sigma_z = (p_0 - p)\,\frac{2b^2(c^2 - a^2)}{\nu(c^2 - b^2)(b^2 - a^2)} \qquad [4]$$

quations [1] and [4] serve to determine σ_z and p as functions of Eliminating σ_z we find

$$\frac{dp}{dz} - \alpha fp = 0 \text{ where } \alpha = \nu\,\frac{c^2 - b^2}{b(c^2 - a^2)}$$

that $p = Ce^{\alpha fz}$. Then Equation [4] gives

$$\sigma_z = \frac{2\pi b}{A\alpha}\,(p_0 - Ce^{\alpha fz})$$

$z = l$, $\sigma_z = 0$ and at $z = 0$, $A\sigma_z = P$.

The first of these conditions gives $C = p_0e^{-\alpha fl}$ and the second en

$$P = p_0\,\frac{2\pi b}{\alpha}\,(1 - e^{-\alpha fl}) \qquad [5]$$

the distribution of pressure is now given by

$$p = p_0e^{-\alpha f(l-z)} \qquad [6]$$

gain some idea of the variations involved, consider the case $= b$, $c = \infty$. Then

$$\alpha fl = \nu\,\frac{c^2 - b^2}{b(c^2 - a^2)}\,fl = \nu f$$

The value of ν will be about 0.3. The value of f will probably be between 0.3 and 1. Taking the latter value so as to get the most extreme variations of p with z, and P with l, we have $\alpha fl = 0.3$. The corresponding variation of p with z is sensibly linear. It would be practically linear for considerably longer joints.

Considering variations of l, keeping $f = 1$ and $\nu = 0.3$, the variation of P with l is found to be linear over the range of practical values of l/b. This agrees with the test results of Ries and Siebel (12, 13, 14).

The case of a push-out test may be obtained from Equations [5] and [6] by reversing the signs of f and P. The results may be written

$$P = p_0\,\frac{2\pi b}{\beta}\,(e^{\beta fl} - 1) \qquad [7]$$

$$p = p_0e^{\beta f(l-z)} \qquad [8]$$

where P is the push-out force and $\beta = \nu(c^2 - b^2)/b(c^2 - a^2)$.

Appendix

The Plasticity Problem

The theoretical results obtained have been based on the plastic-elastic stress curves of Figs. 5 and 6, for pressure applied within a hole in an infinite plate, and the plastic stress curves of Fig. 9.

The equations for the plastic parts of all these curves have a common derivation (A. Nadai, "Plasticity," Chap. 28).[7] The law of plastic flow adopted, that of von Mises and Hencky, is in the general case of three principal stresses s_1, s_2, s_3.

$$(s_2 - s_1)^2 + (s_3 - s_2)^2 + (s_3 - s_1)^2 = 2s_0^2$$

where s_0 is the yield stress in simple tension. In the present problem we have the principal stresses s_r, s_t, radially and tangentially, in the plane of the plate, and the third principal stress is taken as zero. The plasticity condition then reduces to

$$s_r^2 - s_rs_t + s_t^2 = s_0^2 \qquad [9]$$

The curve showing this relation between s_r and s_t is an ellipse. The use of an eccentric angle θ of this ellipse as a parameter, by which the stresses are expressed as

$$s_r = \frac{2s_0}{\sqrt{3}}\sin\left(\theta - \frac{\pi}{6}\right),\quad s_t = \frac{2s_0}{\sqrt{3}}\sin\left(\theta + \frac{\pi}{6}\right) \qquad [10]$$

leaves the plasticity condition automatically satisfied. Both s_r and s_t are limited to lie between $\pm\frac{2s_0}{\sqrt{3}}$ or $\pm 1.15\,s_0$, and this is the limiting radial stress frequently referred to in the paper.

The equation of equilibrium is

$$r\frac{ds_r}{dr} + s_r - s_t = 0$$

Using the variables

$$s = \frac{s_r + s_t}{2} = s_0\sin\theta, \qquad s' = \frac{s_t - s_r}{2} = \frac{s_0}{\sqrt{3}}\cos\theta \qquad [11]$$

this equation is transformed into

$$r\frac{d}{dr}\left[\sin\left(\theta - \frac{\pi}{6}\right)\right] = \cos\theta$$

[7] Ref. (18), Chap. 28, p. 185.

and the integral of this is

$$r^2 = A^2 e^{\sqrt{3}\theta} \sec\theta \quad [12]$$

where A is the arbitrary constant. It is determined as soon as the stress s_r is given at some value of r. For the corresponding value of θ can then be found from Equation [10], and substitution in Equation [12] then gives A.

Curves which yield the solution of all the special problems may be obtained by plotting $\frac{r}{A}$ against θ from Equation [12], and s_r and s_t against θ from Equation [10]. Fig. 14 shows these curves over the range required in the problems of the paper. For a special case, as for instance $s_r = -0.6\,\frac{2s_0}{\sqrt{3}}$ at $r = b$, we have $-\sin\left(\theta - \frac{\pi}{6}\right) = 0.6$ at $r = b$, or at $r/A = b/A$. Picking out the point on the curve of $s_r\sqrt{3}/2s_0$ which has the ordinate 0.6, we find that

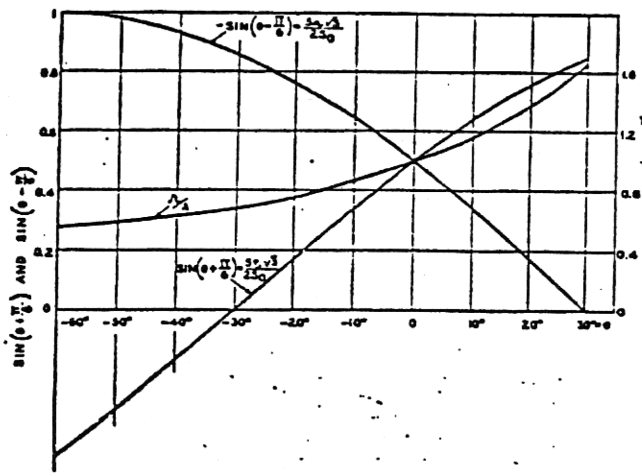


Fig. 14 Special-Problem Curves

it corresponds (for the same θ) to $r/A = 0.9$. Thus $b/A = 0.9$ and $A = 1.115$. The curve of $s_r\sqrt{3}/2s_0$ versus r/A can now be converted into the curve of s_r/s_0 versus r/b, which appears in Fig. 9.

The problem of the infinite plate, with its elastic exterior zone, is solved by observing that since in the elastic zone the stresses s_r and s_t are equal in magnitude, though opposite in sign, the plasticity condition is satisfied at a boundary $r = c$ between the elastic and plastic zones if, from Equation [9] with $s_t = -s_r$, $3s_r^2 = s_0^2$ or $s_r = \pm s_0/\sqrt{3}$. If we then solve the plasticity problem as outlined with the boundary condition $s_r = -s_0/\sqrt{3}$ at $r = c$, the curve of s_r/s_0 can be plotted against r/c. This yields the s_r curve of Fig. 5, AGB, with abscissas r/c. The curve terminates at A, with the limiting stress $1.15\, s_0$ at $r/c = 0.567$. Thus the plate can only be made to yield as far as $c/a = 1/0.567 = 1.75$. Any value of the ratio a/c larger than 0.567 corresponds to the choice of a point between A and B for the hole and the pressure required to cause yield to the corresponding extent, $c - a$, is given by the ordinate.

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